Analytical and experimental evaluation of the stress-strain curves of sheet metals by hydraulic bulge tests

Lucian LĂZĂRESCU[1, a], Dan-Sorin COMŞA[1,b] and Dorel BANABIC[1,c]

[1]Technical University of Cluj-Napoca, Department of Manufacturing Engineering,

B-dul Muncii nr. 103-105, 400641 Cluj-Napoca Romania

[a]lucian.lazarescu@tcm.utcluj.ro, [b]dscomsa@tcm.utcluj.ro, [c]banabic@tcm.utcluj.ro

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Abstract. This paper presents a new methodology for the determination of the biaxial stress – strain curves by hydraulic bulging tests with circular die. In order to validate the methodology, the authors have performed both stepwise and continuous bulging experiments. The pressure, polar height and curvature radius have been measured in different stages of the deformation process or continuously recorded during the test.

Introduction

The successful implementation of the finite element simulation in the design phase of sheet metal forming processes depends on the accuracy of the material characteristics. From this point of view, the hardening law defining the relationship between the flow stress and the plastic strain has a major influence on the quality of the numerical results.

The researchers concerned by the characterisation of the inelastic behaviour of sheet metals have developed several methodologies for the determination of the biaxial stress – strain curves through hydraulic bulging tests. Almost all of these methodologies are based on analytical formulas that define the polar thickness and the curvature radius of the specimen in relation with one or more process parameters. Hill [1] developed an analytical model of the hydraulic bulging process. He admitted the spherical shape of the dome and neglected the influence of the fillet radii located at the entrance of the insert die on the geometry of the specimen. Hill also obtained a formula for the evaluation of the polar thickness from a simple kinematical assumption referring to the evolution of the specimen surface. Panknin [2] also performed experimental studies on the hydraulic bulging. His main interest consisted in the accurate determination of the process parameters. Panknin proposed a formula for the calculation of the curvature radius. This relationship takes into account the effect of the fillet radius on the dimensional characteristics of the dome. By comparing the analytical results with experimental data, Panknin established that the calculated curvature radius deviates with up to 10% from the experimental value in the case of large polar heights. Chakrabarty [3] improved the accuracy of the formulas previously proposed by Hill by taking into account the hardening effects. Gologranc [4] performed both experimental and theoretical studies on the hydraulic bulging. He noticed that the values of the polar thickness predicted by Hill's formula were considerably different from his own experimental results. Shang [5] extended the analytical models developed by Hill in order to take into account the fillet radius of the die insert. According to his experimental observations, the value of the fillet radius has a small influence on the polar strains.

Atkinson [6] also tried to improve the accuracy of the analytical predictions referring to the polar thickness and dome radius. Kruglov [7] developed a simple formula for the calculation of the polar strains. This formula is based on the assumption that the meridian strain is uniformly distributed on the dome surface.

The development of the computers and data acquisition systems made possible important advances in the field of sheet metal testing. In the particular case of the hydraulic bulging, the progress was mainly related to the optical strain measuring systems. These systems consist in CCD cameras controlled by a videogrammetric software [8]. Recently, Koç and his co-workers [9] reported experimental results obtained by using the ARAMIS 3D optical measurement system for the continuous recording of the polar height, curvature radius and polar thickness of the specimens.

The main objective of this paper consists in the development of a new methodology for the accurate and efficient determination of the biaxial stress – strain curves by hydraulic bulging through circular dies. The methodology is based on a modified version of Kruglov'a formula. The modification consists in taking into account the non-uniform distribution of the strains on the specimen surface by means of a correction factor. Both stepwise and continuous hydraulic bulging experiments have been performed in order to validate the methodology. In the case of the continuous tests, the pressure, polar height and curvature radius have been measured using the ARAMIS 3D optical system.

Analytical approach

Figure 1 shows the principle of the hydraulic bulging test. The circular specimen is firmly clamped under the die insert at the top of a hydraulic cylinder. During the bulging process, the sheet metal is deformed by the continuously increasing pressure applied on its bottom surface.

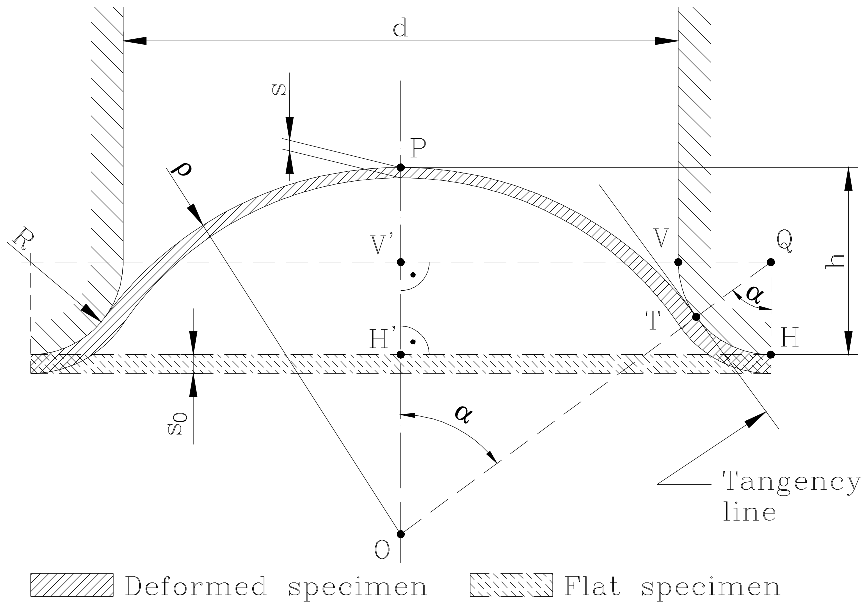

Fig.1. Schematic representation of the specimen subjected to hydraulic bulging

The following points and dimensions shown in Figure 1 are relevant for the subsequent discussion:

O, P – current centre and pole of the dome surface (approximated by a spherical cap)

Q, V, H – centre and ends of the fillet joining the orifice and the bottom surface of the bulging die

T - point where the free profile of the specimen becomes tangent to the fillet arc of the bulging die

V', H' - projections of the points V (Q) and H, respectively, on the vertical axis OP

d - diameter of the bulging orifice

R - fillet radius of the bulging orifice

s_0 - initial (nominal) thickness of the specimen

s - current thickness of the specimen in the polar region (point P)

ρ - current radius of the dome surface

h - height defining the current position of the pole P

α - angle spanned by the arcs $\overset{\frown}{\mathrm{TP}}$ and $\overset{\frown}{\mathrm{TH}}$ respectively (α is expressed in radians).

Both the diameter d and the fillet radius R are constants representing dimensional characteristics of the experimental device. The initial thickness of the specimen s_0 is also constant for a given experiment. All the other quantities mentioned above (s, ρ, h, and α) are variables depending on the current level of the pressure (p) acting on the bottom surface of the sheet metal.

One assumes that the mechanical response of the specimen is similar to that of a membrane in plane-stress conditions. Under such circumstances, the load state in the polar region can be assimilated to the biaxial traction. The current value of the biaxial surface stress (σ_b) is defined by Laplace's formula

$$\sigma_b = \frac{p\rho}{2s}, \tag{1}$$

while the corresponding thickness strain (the so-called biaxial strain ε_b) can be evaluated using the relationship

$$\varepsilon_b = \ln\frac{s_0}{s}. \tag{2}$$

Eqs (1) and (2) can be used to obtain a biaxial stress - strain diagram only if the quantities p, ρ and s are either measured or derived from other experimental data. The current level of the pressure p can be easily measured using a sensor connected to the hydraulic chamber of the experimental device. The other process variables, namely the curvature radius ρ and the polar thickness s are less accessible to the direct determination. It is more convenient to obtain their values in an indirect manner, using approximate formulas that involve the current value of the polar height h.

The curvature radius ρ can be evaluated with Panknin's formula [2]

$$\rho = \frac{1}{2h}\left(\frac{d}{2}+R\right)^2 + \frac{h}{2} - R. \tag{3}$$

The experimental studies performed by other researchers [9] proved that amongst the numerous relationships that can be used to compute the current value of the polar thickness s Kruglov's formula [7] provides the best results. This relationship is based on the volume preservation hypothesis. At the level of the polar region, such a constraint can be written as follows:

$$\varepsilon_b = 2\varepsilon_m, \tag{4}$$

where ε_m is the meridian strain. According to Kruglov's approach, one assumes that ε_m can be calculated using the approximation (see also Figure 1):

$$\varepsilon_m = \ln\frac{\overset{\frown}{\mathrm{TP}} + \overset{\frown}{\mathrm{TH}}}{\mathrm{HH'}}. \tag{5}$$

Eq (5) would be rigorously valid only if the meridian strain were uniformly distributed on the dome surface. In general, this is not true, because the local thickness of the specimen gradually decreases from the clamping contour towards the pole. A modified version of Kruglov's formula will be presented at the end of this analysis with the aim of improving its accuracy.

Using the notations shown in Figure 1, the following expressions of the quantities involved in Eq (5) can be written:

$$\overset{\frown}{\mathrm{TP}} = \rho\alpha, \quad \overset{\frown}{\mathrm{TH}} = R\alpha, \quad \mathrm{HH'} = \frac{d}{2} + R, \tag{6}$$

where (see also Eq (3))

$$\sin\alpha = \frac{\mathrm{V'Q}}{\mathrm{OQ}} = \frac{\frac{d}{2}+R}{\frac{1}{2h}\left(\frac{d}{2}+R\right)^2+\frac{h}{2}}, \tag{7}$$

i.e.

$$\alpha = \arcsin\frac{\frac{d}{2}+R}{\frac{1}{2h}\left(\frac{d}{2}+R\right)^2+\frac{h}{2}}. \tag{8}$$

As a consequence, Eq (5) becomes:

$$\varepsilon_m = \ln\frac{\alpha}{\sin\alpha}. \tag{9}$$

After replacing ε_m defined by Eq (9) into Eq (4), one obtains the following expression of the polar biaxial strain:

$$\varepsilon_b = 2\ln\frac{\alpha}{\sin\alpha}. \tag{10}$$

Eq (10) is the formula proposed by Kruglov. When combined with Eq (2), this relationship allows the evaluation of the current polar thickness:

$$s = s_0\exp\left(-\varepsilon_b\right) = s_0\left(\frac{\alpha}{\sin\alpha}\right)^{-2}. \tag{11}$$

One may notice that Eqs (10) – (11) involve only the measurement of the dome height h. As mentioned above, the studies performed by other researchers have proved that these relationships are in good agreement with the experimental data. The accuracy of Kruglov's formula is still improvable if Eq (5) is modified as follows:

$$\varepsilon_m = \left(1+c\,\alpha\right)\ln\frac{\overset{\frown}{\mathrm{TP}}+\overset{\frown}{\mathrm{TH}}}{\mathrm{HH'}}. \tag{12}$$

The coefficient c is a strictly positive constant that takes into account the non-uniformity of the meridian strain distribution on the dome surface. The determination of the parameter c will be detailed at the end of this discussion.

By taking into account Eqs (6) and (8), it is possible to rewrite Eq (12) in the form

$$\varepsilon_m = \left(1+c\,\alpha\right)\ln\frac{\alpha}{\sin\alpha}. \tag{13}$$

After replacing ε_m defined by Eq (13) into Eq (4), one obtains the following expression of the polar biaxial strain:

$$\varepsilon_b = 2\left(1+c\,\alpha\right)\ln\frac{\alpha}{\sin\alpha}. \tag{14}$$

The difference between Eq (14) and Kruglov's formula (see Eq (10)) consists only in the presence of a correction factor involving the constant parameter c. When combined with Eq (2), this relationship allows the evaluation of the current polar thickness:

$$s = s_0 \exp(-\varepsilon_b) = s_0 \left(\frac{\alpha}{\sin\alpha}\right)^{-2(1+c\alpha)} . \tag{15}$$

The practical use of Eqs (14) and (15) is conditioned by the determination of the coefficient c. This parameter can be easily established if the final value of the polar thickness s_{min} is measured after removing the specimen from the hydraulic bulging device. The thickness determination can be performed using a measuring device that comes into contact with the upper and lower surfaces of the specimen. Let h_{max} and α_{max} be the polar height and the angle spanned by the dome surface, respectively, both quantities corresponding to the final stage of the bulging process. In this configuration of the specimen, Eq (15) can be written as follows:

$$s_{min} = s_0 \left(\frac{\alpha_{max}}{\sin\alpha_{max}}\right)^{-2(1+c\alpha_{max})} , \tag{16}$$

where (see Eq (8))

$$\alpha_{max} = \arcsin \frac{\frac{d}{2}+R}{\frac{1}{2h_{max}}\left(\frac{d}{2}+R\right)^2 + \frac{h_{max}}{2}} , \tag{17}$$

Eq (16) leads to the following formula for the evaluation of the coefficient c:

$$c = \frac{\ln\sqrt{\frac{s_0}{s_{min}}} - \ln\frac{\alpha_{max}}{\sin\alpha_{max}}}{\alpha_{max}\ln\frac{\alpha_{max}}{\sin\alpha_{max}}} . \tag{18}$$

Experimental procedure

The hydraulic bulge tests have been performed using the experimental device shown in Figure 2. The main components of the system are the bulging cylinder, the hydraulic device and the optical measurement system ARAMIS. The circular specimens have been cut from DC04 steel sheets with the nominal thickness of 0.85 mm.

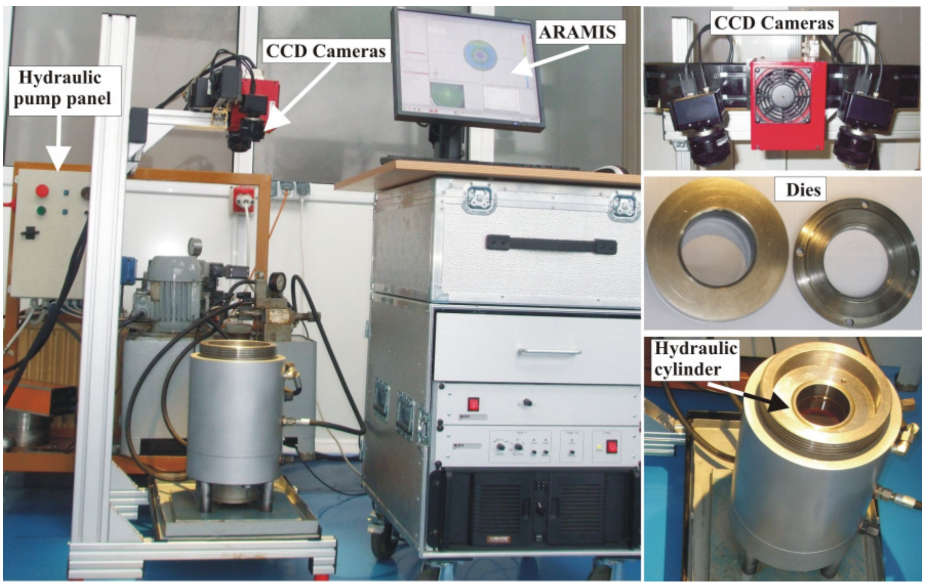

Fig. 2. Experimental set-up for the hydraulic bulge test

Two types of experiments have been performed for the validation of the methodology proposed in this paper. The first category of tests consists in stepwise bulging experiments. In the case of these tests, the straining process has been stopped at different values of the polar height. The corresponding level of the bulging pressure has been also recorded. After removing the specimen from the bulging device, the deformed surface has been inspected with a 3 D Coordinate Measuring Machine, with the aim of determining its minimum thickness, polar height and curvature radius. These experimental results have been used for the calculation of discrete points belonging to the biaxial stress – strain curve.

The second type of tests consists in continuous bulging experiments using a 3D optical measurement system ARAMIS. In this case, the evolution of the pressure and polar height has been recorded by a pressure sensor and two CCD cameras, respectively. The experimental data has been used for the determination of biaxial stress – strain curves.

Experimental results

Bulge radius and dome height

The dome radius corresponding to each stage of the bulging process has been evaluated using Eq (3) and the current value of the polar height measured with the ARAMIS system. In the case of the stepwise experiments, both the radii and the polar heights have been determined on a 3D Coordinate Measuring Machine. The maximum deviation from the spherical shape, as established by 3D CMM, is 0.1137 mm, while the minimum deviation is 0.0615 mm. Figure 3.a shows the variation of the bulge radius as a function of the polar height given by Eq (3), as well as the corresponding data obtained from the stepwise experiments. One may notice that the predictions of Eq (3) are closed to the experimental data.

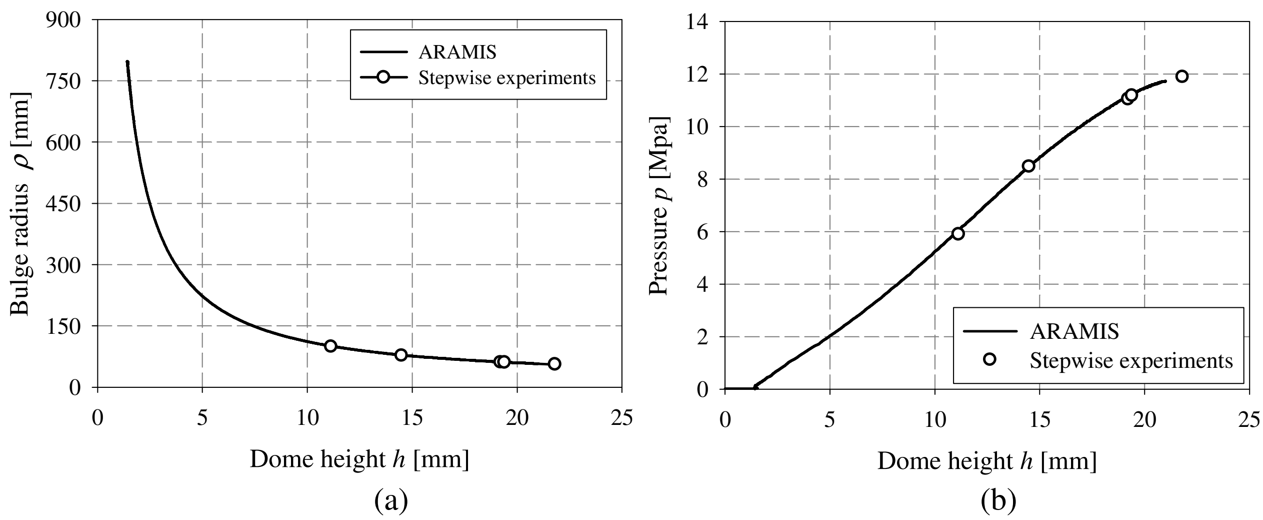

Fig. 3. Bulge radius vs dome height (a); Hydraulic pressure vs dome height (b)

Figure 3.b shows the variation of the pressure as a function of the dome height. The experimental results have been obtained by continuous measurement with the ARAMIS system. The diagram also contains the discrete values provided by the 3D CMM in the case of the stepwise experiments. As one may notice from Figure 3.b, there are no significant differences between the continuous and stepwise measurements. The maximum pressure attained during the stepwise bulging tests has been 11.87 MPa. The value of the polar height corresponding to this load has been 21.81 mm.

Polar thickness

Figure 4 shows the variation of the polar thickness as a function of the polar height. The data presented on the diagram has been obtained both by measurement and calculations. As noticeable from Figure 4, up to a polar height of about 5 mm, both curves obtained from the standard and modified Kruglov's formulas are almost coincident with the data provided by the ARAMIS system.

For larger values of the polar height, the variation predicted by the standard Kruglov's formula gradually deviates from the experimental curve, while the predictions of the modified formula remain in a closer neighbourhood. The results of the stepwise experiments are also in better agreement with the data provided by the ARAMIS system and the predictions of the modified Kruglov's formula.

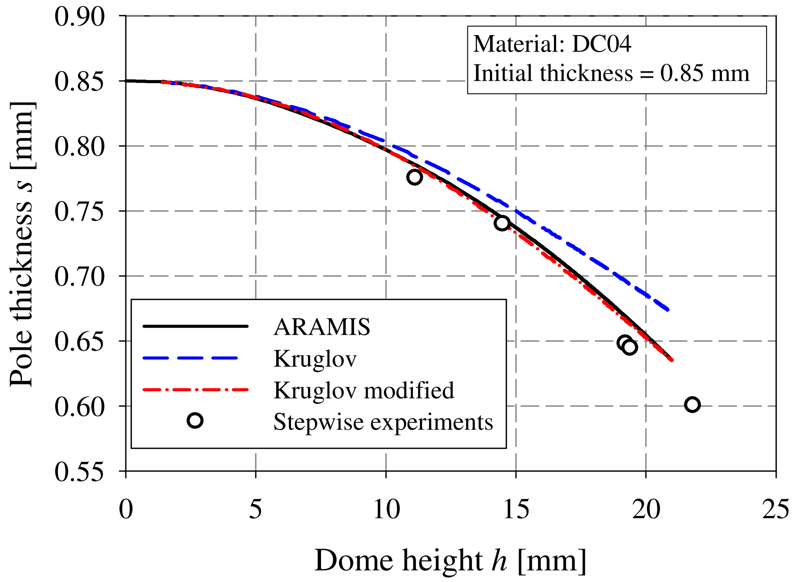

Fig. 4. Polar thickness vs dome height

Biaxial stress – strain curves

A similar comparison has been performed in the case of the biaxial stress – strain curves. As noticeable from Figure 5, the results obtained using the Kruglov's modified formula are in better agreement with the experimental data provided by the ARAMIS system. The accuracy of the predictions remains very good up to the end of the experimental curves. In contrast, the standard Kruglov's formula tends to underestimate both the biaxial strains and stresses in the final stages of the bulging experiment (see the detail in Figure 5). The correction included in the modified relationship is able to drag the calculated curve closer to the experimental data provided by the ARAMIS measurement system. The same conclusion can be drawn when comparing the predictions of the modified Kruglov's formula with the results obtained from stepwise experiments.

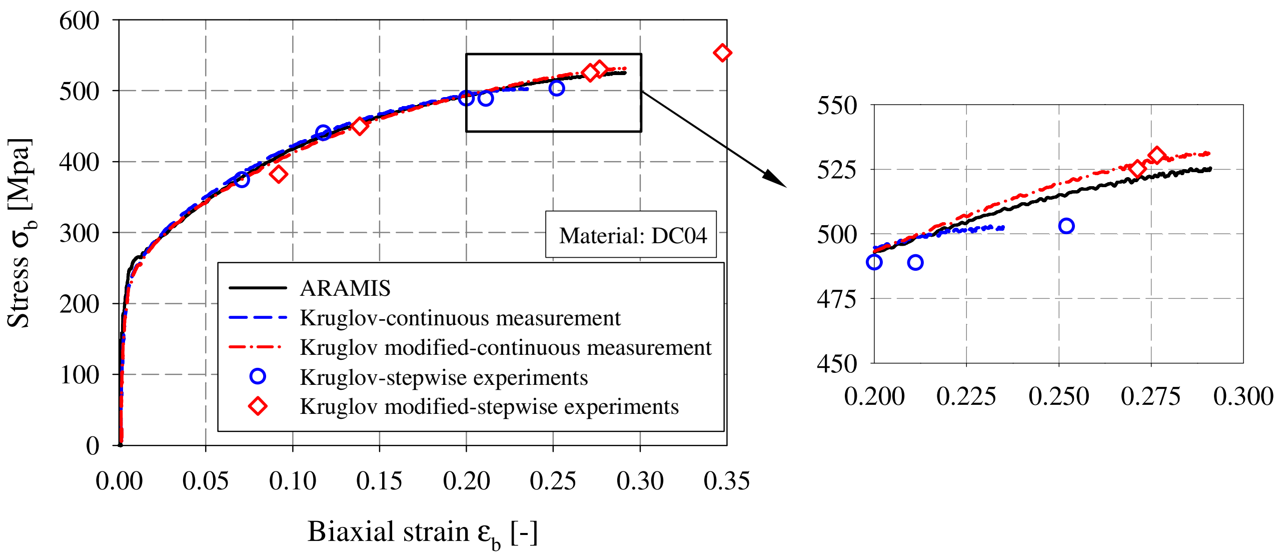

Fig. 5. Biaxial stress-strain curves

Conclusion

The paper presents a new methodology for the experimental determination of the biaxial stress – strain curves. The methodology operates with a modified version of Kruglov's formula that takes into account the non-uniformity of the strain distribution on the dome surface. The modification is performed by means of a correction coefficient *c* related to the polar thickness of the specimen in the final stage of the bulging process. The comparison with experimental data provided by the ARAMIS system shows an improved accuracy of the modified Kruglov's formula. The most important advantage of the methodology presented in this paper consists in its simplicity. The experimental measurements can be performed using general purpose devices (pressure and displacement sensors). Due to its accuracy, the methodology can be easily implemented in the industrial laboratories involved in the testing of sheet metals.

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